Inference at $*$ 1
of proof for Lemma connex_iff_trichot:

$\vdash \forall T$:Type, R:($T \to T \to \mathbb{P}$).
($\forall a$, b:T. Dec($R(a,b)$))
$\Rightarrow$ (Connex(T;x,y.$R(x,y)$)
$\iff$ ($\forall a$, b:T.
strict_part(x,y.$R(x,y)$;a;b)
$\vee$ Symmetrize(x,y.$R(x,y)$;a;b)
$\vee$ strict_part(x,y.$R(x,y)$;b;a)))
by ((((Unfolds “connex strict_part symmetrize“ 0)
CollapseTHEN (GenRepD))·)

CollapseTHENA ((Auto_aux (first_nat 1:n) ((first_nat 1:n),(first_nat 3:n)) (first_tok :t) inil_term)))·

1:

1. T : Type
2. R : $T \to T \to \mathbb{P}$
3. $\forall a$, b:T. Dec($R(a,b)$)
4. $\forall x$, y:T. $R(x,y) \vee R(y,x)$
5. a : T
6. b : T

$\vdash$ ($R(a,b)$ & ($\neg R(b,a)$)) $\vee$ ($R(a,b)$ & $R(b,a)$) $\vee$ ($R(b,a)$ & ($\neg R(a,b)$))

2:

1. T : Type
2. R : $T \to T \to \mathbb{P}$
3. $\forall a$, b:T. Dec($R(a,b)$)
4. $\forall a$, b:T. ($R(a,b)$ & ($\neg R(b,a)$)) $\vee$ ($R(a,b)$ & $R(b,a)$) $\vee$ ($R(b,a)$ & ($\neg R(a,b)$))
5. x : T
6. y : T

$\vdash$ $R(x,y) \vee R(y,x)$

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